

## Learning Mathematics with tools

**M. G. Bartolini Bussi (Università di Modena), G. Chiappini (ITD, CNR Genova),**

**D. Paola (liceo Issel, Finale Ligure), M. Reggiani (Università di Pavia),**

**O. Robutti (Università di Torino), ITALY**

### Summary

*This paper presents the most recent researches in Mathematics Education in Italy, dealing with the use of tools and technologies at school. Its origin is in the document of Unione Matematica Italiana (U.M.I., the association of school and university teachers of Mathematics), which has published a curricular project for school Mathematics from the 1<sup>st</sup> to the 12<sup>th</sup> grade. The paper is a part of the book that is presented at ICME as the collection of all the most recent Italian researches in Mathematics Education.*

### Introduction

Learning Mathematics with tools, which can be traditional or more advanced, is simpler than in an abstract way (called symbolic-reconstructive by the Psychologists): it is a perceptive-motor way of learning, because it is grounded in actions, perceptions, and reactions due to the feedback received in the use of tools (Antinucci, 2001).

The methodologies, the activities, the curricular sequences used by the teacher are different from a traditional way of teaching, based only on frontal lessons, and the role of teacher is very important not only in realising students' activities, but also in planning them.

This paper is related with the theme of the DG20, because it offers an overview on Italian recent researches on the use of tools in teaching and learning Mathematics. So, it is aimed at giving a possible answer to some of the questions asked by the DG:

1. About the Theme (C) Tools and Technologies:
  - What are appropriate/meaningful uses of technology for upper secondary mathematics?
  - What can be the different roles of tools and technologies in the mediation of learning?
  - How can the use of tools and technologies influence students' cognitive processes?
2. About the theme (D) Curriculum:
  - Can new theoretical trends influence school curricula?
  - What are the new curricular trends recently developed in different countries?

### Tools in Mathematics Education: recent Italian trends

A lot of research studies have been carried out in the last years concerning learning with tools, i.e. learning in an environment that is richer than the standard paper and pencil one. The empirical and theoretical studies of Italian researchers have led to the elaboration of the idea of mathematical

laboratory (e.g. Mariotti, 2002; Chiappini & Reggiani, 2003; Arzarello, Paola & Robutti, 2002; Arzarello, Andriano et al., 2000; Bonotto et al, 2002) and have been collected by the committee appointed by the U.M.I. for the production of new curricula between the years 2000 and 2003. In the following we shall quote wide excerpts of the document prepared by the committee of the U.M.I.<sup>1</sup>, together with some further elaborations of the authors. The U.M.I. text has been partially adopted in the official documents of the Italian Ministry of Education<sup>2</sup>.

*'A mathematics laboratory is not intended as opposed to a classroom, but rather a methodology, based on various and structured activities, aimed to the construction of meanings of mathematical objects. A mathematics laboratory activity involves people (students and teachers), structures (classrooms, tools, organisation and management), ideas (projects, didactical planning and experiments). We can imagine the laboratory environment as a Renaissance workshop, in which the apprentices learned by doing, seeing, imitating, communicating with each other, in a word: practicing. In the laboratory activities, the construction of meanings is strictly bound, on one hand, to the use of tools, and on the other, to the interactions between people working together (without distinguishing between teacher and students). It is important to bear in mind that a tool is always the result of a cultural evolution, and that it has been made for specific aims, and insofar, that it embodies ideas. This has a great significance for the teaching practices, because the meaning can not be only in the tool per se, nor can it be uniquely in the interaction of student and tool. It lies in the aims for which a tool is used, in the schemes of use of the tool itself. The construction of meaning, moreover, requires also to think individually of mathematical objects and activities'*<sup>1</sup> (p. 32).

This introduction is accompanied by an annotated list of exemplary tools, taken from everyday experience, and advanced technological tools as well.

*'Poor materials. For example, working with transparent slides, the crease of paper, the use of pins, grid paper, should not only be considered an activity specifically designed for pupils of primary schools, but it could be a meaningful starting point for mathematical activities at different levels. Furthermore, the use of poor materials, made by the students themselves, represents a significant activity, in the spirit of the Renaissance workshops'*<sup>1</sup> (p. 32).

Some research groups have designed, carried out and analysed teaching experiments (usually in grades 1<sup>st</sup>-8<sup>th</sup>) concerning either mathematical objects built with poor materials (e.g.) Facenda et al., 2002) or everyday objects analysed with mathematical lenses, in order to elicit the implicit mathematical ideas (e. g. cardboard area units, an informational booklet issued by "Poste Italiane", cotton tips and square napkins in Bonotto & Ceroni, 2003; supermarket receipts in Bonotto, 2001;

---

<sup>1</sup> <http://www.dm.unibo.it/umi/italiano/Didattica/2003/secondaria.pdf>

<sup>2</sup> [http://www.istruzione.it/news/2002/allegati/sperimentazione/profilo\\_terminale.pdf](http://www.istruzione.it/news/2002/allegati/sperimentazione/profilo_terminale.pdf)

TV schedule in Bonotto, 2003). In the latter case the introduction of cultural artefacts from out-of-school experience aims at creating a new tension between school mathematics and everyday-life knowledge. The shift from the study of everyday gears to mathematical objects like circles has been studied by Bartolini Bussi, Boni & Ferri, to appear). The analysis of a traditional mathematics object like the graduated ruler for the introduction of decimal numbers and their properties is in Bonotto et al. (2002). The ruler suggests the shift towards tools that have been created and constructed in history for specific mathematics purposes, such as the mathematical machines.

*'The mathematical machines. The possibilities offered by the mathematical machines, of manipulating objects physically, as in the case of machines generating conics, often induces exploration and construction of mathematical meanings, different but not less interesting than the one offered by a Dynamic Geometry Software'*<sup>1</sup> (p. 32).

A mathematical machine (related to the geometry field) has a basic aim, that does not depend on the practical use (if any) of the artefact. It aims at forcing a point, a line segment or a plane figure (supported by a suitable material support that makes them visible and touchable) to move or to be transformed according to a mathematical law that has been determined by the designer. A very large collection of mathematical machines, used in exhibition and in the classroom as well, is in Modena ([www.mmlab.unimore.it](http://www.mmlab.unimore.it)). The most well-known mathematical machine is the compass (see Bartolini Bussi & Boni, 2003; Bartolini Bussi, Boni & Ferri, to appear), that is the ancestor of many curve drawing devices (Bartolini Bussi, 2001; several papers by Pergola et al.). Another class of mathematical machines is given by perspectographs (Bartolini Bussi, Mariotti & Ferri, 2003) that are related to the ancient 3d theory of conics (Bartolini Bussi, to appear). An international review of ancient instruments that can be used in the classroom is in Bartolini Bussi (2000). Besides geometrical ones, there are also instruments for arithmetics (for a didactical analysis of abacus see Bartolini Bussi & Boni, 2003; Ferri 2002, Betti & Canalini 2002). The theoretical framework of the above experimental studies is based on the Vygotskian idea of semiotic mediation.

*'The DGE: Dynamic Geometry Software. During the last years, the teaching of Geometry has been supported by the introduction of Dynamic Geometry Software, that are microworlds designed for specific educational tasks. They allow the students to experience, to explore and to observe, in order to look for invariants, patterns and regularities, and to formulate conjectures, and to test them in the software itself. In such a kind of interaction with the microworld, the student can meet the knowledge embodied in the software, and he can then construct a proper geometric knowledge. From this activity, the student can persevere, a more theoretical knowledge, namely the proof, as the activity aimed at justifying why a certain property holds in a given theory'*<sup>1</sup> (p. 32).

This is a very popular field. Many Italian researchers have published papers concerning classroom experiments on Cabri. The approach to the definition of geometrical figures is considered by Pesci (2000), with possible extension to primary school pupils too. However most of researchers are interested in analysing the processes of conjecturing and proving (Olivero, 2001, 2003; Olivero et al. 2002, Arzarello et al., 1999a, 1999b; Olivero, Paola & Robutti, 2001). In particular, Arzarello (2000) describes the learning of proof as a long process of interiorisation, through specific and complex mental dynamics of pupils, from perceptions and actions within technological environments towards structured abstract mathematical objects, embedded in a theoretical framework: the main issues in the analysis of students' performances consist in metaphors, deictics, mental times, narratives, functions of dragging, abductions, linear vs. multivariate language and so on, to be used within an embodied cognition perspective. Arzarello, Olivero et al. (2002) offer a fine grain analysis of the process of dragging in conjecturing and proving. Mariotti (2001a, 2001b) analyses the teacher's role: assuming a Vygotskian perspective, attention is focussed on the social construction of knowledge and on the semiotic mediation accomplished through cultural artefacts. The functions of specific elements of the software are described and analysed as instruments of semiotic mediation used by the teacher in classroom activities.

Another group of papers consider the measuring process in the Cabri microworld. Olivero & Robutti (2001a, 2001b, 2002) study the shift from perception to theory and back again fostered by the measure tool and the effectiveness of this shift in the construction of a proof, after the conjecturing phase.

Two papers (Laborde & Mariotti, 2002; Mariotti, Laborde & Falcade, 2003) study the approach to the concept of function in the Cabri microworld. The authors analyse experiments in secondary school where the dynamic features provide a basic representation of both variation and functional dependency.

The comparison between Mascheroni geometry and Cabri geometry is studied by Galoppin P. & Zuccheri L. in secondary school. The difficulties met by students from abroad in the Cabri microworld are studied by Rocco (2000).

Accascina & Margiotta in a set of papers discuss interesting problems on the geometry of triangle to be used with secondary school students. Bernardi (2003) discusses some epistemological issues related to dragging in Cabri. The role of new technologies in the teaching of geometry is analysed, in a context of teacher training, by De Petro et al (2003) and Zuccheri (2003).

*'The CAS: Computer Algebra Systems. In the teaching of algebra and calculus a primary role is represented by the CAS, which have different integrated environments, generally the numerical, the graphical, the symbolic and the programming. The introduction of CAS in the teaching of algebra*

and calculus permits to circumscribe the use of symbolic calculation with paper and pencil only to the simpler cases, in order to let the more complex calculations be done by the student with the aid of the software. From a didactical point of view, we can have a double advantage, because the student is free to concentrate on the meaning of the calculation, if he can devolve the difficult one to the CAS. Also the CAS, as the DGE, offers to the students different environments in which he can explore and do conjectures, to carry out the construction of meaning of mathematical objects. Last, but not least, the programming language offered by CAS is particularly useful in the consolidating of the concept of function, variable, input and output values, and of the data collecting (list, array, matrix, ...)'<sup>1</sup> (p. 33).

Many researchers confronted the theme of the didactical use of the computer algebra systems but the majority of the studies about this topic was carried out using graphic-symbolic calculators (see below).

The most diffuse CAS in Italy at the didactic level is *Derive*, in a version for Windows, in that it is particularly simple to use and, moreover, it is available in Italian. There are many didactical proposals and suggestions in the school text-books, but less numerous are the research activities. The use of *Derive* as a symbolic manipulator in the phase of approach to algebra is studied in Reggiani 2002a, where some aspects of the problem of writing, reading and processing algebraic expressions with *Derive* are confronted, focusing on abilities required and promoted by this software and on differences with paper and pencil.

In other studies the use of *Derive* as a help for the formulation of conjectures and as a tool for their verification is confronted. This aspect is proposed in Reggiani 2002b about some questions of divisibility and other problems whose generalization requires algebraic competencies, and in Reggiani 2000 about the study of functions depending on parameters. The role of mediation of the software in the construction of the meaning of parameter through the observation of the graphs and the algebraic manipulation is pointed out.

The strategies used in solving geometric problems in dependence on the tools which can be used are studied by Accascina (2001). The author compares the strategies used by students working with or without *Derive* and analyses the pro and the cons of the use of *Derive* in solving geometric problems in the school final examinations.

A more general question is proposed in Impedovo (2002) who starting from the hypothesis that students have at their disposal all the time (during classes, while studying at home and for any assignment and examination) a Computer Algebra System or, more generally, mathematics software like *DERIVE*, *MAPLE*, *MATHCAD*, or graphic and symbolic calculator, examines in

which way should contents, teaching of mathematical objects, problems, exercises and finally evaluation instruments be modified.

The teaching of algebra and arithmetics (in previous years) is approached also by means of specific microworlds. In the domain of Arithmetic two papers (Bottino, & Chiappini, 2002; Bottino, 2000) deal with the relationship between the use of microworlds and construction of educational environments able to foster teaching and learning processes in this domain. In the domain of Algebra some papers (Chiappini, Pedemonte, & Robotti, 2003; Mariotti, & Cerulli, 2001; Mariotti, & Cerulli, 2002; Cerulli, & Mariotti, 2003) deal with the role of specific microworlds in the learning of Algebra according to an innovative educational approach in which algebraic manipulation is viewed as a demonstration of the equivalence of two form of expressions. It is important to observe that the microworlds described in these papers (the microworlds of ARI-LAB-2 and the microworld named Algebrista) are developed by the authors of these papers.

*'The spreadsheets. The spreadsheets, developed as tools for business and financial calculation, not for educational purpose, have various applications in the school, particularly related to statistics (data collection, organisation, graphical representation, ...) and probability. But another fundamental use of spreadsheets is the one related to modelling, representing functions and even geometric transformations'*<sup>1</sup> (p. 33).

Italian researchers have also studied the role of spreadsheet in the construction of mathematical knowledge. In particular a paper (Arzarello, Bazzini & Chiappini, 2002) analyses the role of a spreadsheet in structuring a didactic space-time of production and communication (SP) able to favour the production and interpretation of formula in the approach to algebra and the use of variables and parameters in modelling complex situation. Through a comparison between the SP structured with the mediation of a spreadsheet and the traditional SP based on the use of paper and pen, this publication suggests a model to analyse algebraic thinking and to design didactic situations apt to build up a genuine algebraic knowledge

Another paper (Lemut, 2003) analyses the role of a spreadsheet in supporting and creating the conditions for Systemic Thinking development. In this paper Systemic Thinking is considered as a general philosophy that, by suggesting a "thinking globally, but acting locally" approach, can represent a major paradigm shift in how we view the world.

*'The symbolic-graphic calculators. All the support environments offered by the software previously described, can be found in the symbolic-graphic calculators, which can be used with more flexibility and simplicity, both for the space occupied, and the time utilised (to move the students from a classroom to a laboratory). Many of these calculators offer the possibility to connect with a*

*sensor, to measure a physical quantity and to collect data in real time. This modality is of particular importance, as far as this is concerned: the possibility of describing a phenomenon into mathematical language, obtaining a model'*<sup>1</sup> (p. 33).

Various researches were carried out in recent years, about the introduction of mathematical concepts through experiments involving perceptuo-motor activities, as for example body motion or the motion of objects, as toys, balls and so on. The didactical aim of these researches is the construction of the meaning of graphs and number tables related to the motion activity, in order to avoid the most frequent misconceptions witnessed in literature. The research aim is to analyse the students' cognitive processes, in terms of a detailed outline of gestures, metaphors and language. These researches started from an initial study of students' performances, analysed through the theory of *embodied cognition*. Their cognitive activities, revealed by words and gestures, are crucial for the genesis of their mathematical understanding. Specifically, the so called *grounding metaphors* and *fictive motions* are cognitive pivots which trigger and support the transition from the empirical and perceptive facts to a more theoretical frame (Arzarello & Robutti, 2001).

Different research studies have been carried out within this framework.

A first set of studies aimed at the construction of the concept of function as a tool for modelling motion with 9<sup>th</sup> graders (Ferrara & Robutti 2002a; Ferrara, & Robutti 2002b; Arzarello, Pezzi & Robutti, 2003). The topic has been explored deeply through various activities, also in environments outside school, as for example Luna-park. These activities that can help and support students in a meaningful approach to algebraic rules, symbols and relationships. The focus is on developing the symbol sense, as well as interconnecting the syntactic and semantic aspects.

Another set of studies carried out with symbolic-graphic calculators, refers to the construction of the concept of integral, starting from approximate measures of areas of figures. This study, based on the didactical aim of introducing Calculus concepts grounding on David Tall's cognitive roots, has the research aim of analysing the students' cognitive processes, in terms of the mediation of technology and gestures, metaphors and language. A long teaching experiment in upper secondary school (11<sup>th</sup>-12<sup>th</sup> grade) is presented (Robutti & Sabena, 2003). The study refers specifically to analyse the passage from finite sums to infinite ones, with the mediation of the technology (Robutti, 2003). The other side of the coin is the concept of derivative, constructed from the local slope of the graph of a function: a fine study based on the use of Zoom in the symbolic-graphic calculators ha carried out in a PhD Thesis and the papers related to it (Maschietto, 2002; Accomazzo & Maschietto, 2002).

Here the list of studies related to the UMI document ends. In the same spirit we may add other tools that have been used in the classroom to enhance mathematics activity.

*Videotapes may be used in the classroom to foster metacognitive activity: students may observe themselves at work and reflect on their processes.*

*In the paper (Furinghetti, Olivero & Paola, 2001) videotapes are used to encourage students to reflect on their reasoning. The same approach appears in the paper (Olivero, Paola & Robutti, 2002). Videotapes may be used also by teachers and by researchers to analyse students' performances.*

There are some experiences of e-learning, as for example the one described in the paper (Iozzi 2002, Osimo 2002), which presents an undergraduate mathematics course at a university of business administration. The course, which is part of a three-years "Degree in Economics of International Markets and New Technology", deals with topics of pre-calculus, calculus, linear algebra. The use of technological tools seems to be essential for today's learning methodologies. The paper offers a challenge to the possibility of changing both the ways of teaching and the contents of a mathematical course.

The introduction of *tools for textbook analysis* in the context of teacher training is studied by Formica et al. (2001).

It seems important to remind that most of the studies reviewed (the ones concerned with classroom activity) focuses not only on the features of the tool, but also on the quality of interaction (student–instrument; student–student; students–teacher). This shared idea has been taken also in the U.M.I. document:

*'The construction of meaning with a methodology based on the Mathematics laboratory is strictly connected with the social interaction of the students, during an activity, carried on in small groups work. During the group activity, the students can share the process of conceptualisation, through a collaborative o cooperative interaction. After the group activity, it is hopeful that a class discussion, led by the teacher, permits the students to share the results of the groups. A mathematical discussion consists in a social interaction aimed at the construction of a common knowledge in the classroom, shared by all the students'*<sup>1</sup> (p. 34).

## References

- Accascina G.: 2001, 'Agli esami con il calcolatore', in *Problem Solving e Calcolatore* (G.Accascina, G.Margiotta, G. Olivieri eds), Franco Angeli Editore, Milano, 211 – 223
- Accascina G., Margiotta G.: 2002, 'Alla ricerca di triangoli equilateri', *Progetto Alice*, prima parte 8, 2002, 175 – 199, seconda parte, 9, 2002, 383 – 408, terza parte, 10, 2003, 37 - 61
- Accomazzo, P. & Maschietto, M.: 2002, 'La transition algèbre/analyse au lycée dans l'environnement des calculatrices graphiques et formelles : quelques éléments', in L.Bazzini, C.Whybrow Inchley (eds), *Actes de la C.I.E.A.E.M. 53 - Verbania*, 339-345.



- Antinucci, F.: 2001, *La scuola si è rotta*, Laterza, Bari.
- Arzarello, F.: 2000, 'Inside and Outside: Spaces, Times and Language in Proof Production', *Proceedings of PMEXXIV*, Hiroshima, Japan, 1, 23-38.
- Arzarello F., Bazzini L. & Chiappini G.: 2002, 'Le pensée algébrique dans une perspective sémiotique. L'environnement du tableur', *Sciences et techniques éducatives*, Vol 9, n°1-2, 219-239.
- Arzarello, F., Olivero, F., Paola, D. & Robutti, O.: 1999, 'Dalle congetture alle dimostrazioni. Una possibile continuità cognitiva', *L'insegnamento della matematica e delle scienze integrate*, vol.22B, N.3, 209-233.
- Arzarello, F., Olivero, F., Paola, D. & Robutti, O.: 1999, 'I problemi di costruzione geometrica con l'aiuto di Cabri', *L'insegnamento della matematica e delle scienze integrate*, vol.22B, N.4, 309-338.
- Arzarello, F., Andriano, V., Olivero, F. & Robutti, O.: 2000, 'Abduction and conjecturing in mathematics', *Philosophica*, 1998, 1, vol.61, 77-94.
- Arzarello, F. & Robutti, O.: 2001, 'From Body Motion to Algebra through Graphing', in H. Chick, K. Stacey, J. Vincent & J. Vincent (eds.), *12th ICMI Study Conference: The Future of the Teaching and Learning of Algebra*, Melbourne, Australia, December 9-14, 2001, vol.1, 33-40.
- Arzarello, F., Olivero, F., Paola, D. & Robutti, O.: 2002, 'A cognitive analysis of dragging practises in Cabri environments', *Zentralblatt für Didaktik der Mathematik*, Vol.34 (3).
- Arzarello, F., Paola, D. & Robutti, O.: 2002, 'Reform project for mathematics in compulsory school in Italy', L Bazzini & C. Whybrow Inchley (eds.), *Proceedings of CIEAEM 53*, Verbania, Italy, 92-102.
- Arzarello, F. & Robutti, O.: 2003, 'Approaching algebra through motion experiences', in Perceptuo-motor Activity and Imagination in Mathematics Learning, Research Forum 1, *Proceedings of PME 27*, Honolulu, July 2003, 1, 111-115.
- Arzarello, F., Pezzi, G. & Robutti, O. (2003). 'Modelling Body Motion: an approach to functions using measure instruments', *Proceedings of 14th ICMI Study Conference*.
- Bartolini Bussi, M. G.: 2000, 'Ancient Instruments in the Mathematics Classroom', in Fauvel J. & van Maanen J. (eds), *History in Mathematics Education: The ICMI Study*, 343-351, Kluwer Academic Publishers.
- Bartolini Bussi, M. G.: 2001, 'The Geometry of Drawing Instruments: Arguments for a didactical Use of Real and Virtual Copies', *Cubo Matematica Educacional*, 3 (2), 27-54.
- Bartolini Bussi, M. G. & Boni M.: 2003, 'Instruments for semiotic mediation in primary school classrooms', *For the Learning of Mathematics*, 23 (2), 12-19.
- Bartolini Bussi, M. G., Mariotti, M. A. & Ferri, F.: 2003, 'Semiotic mediation in the primary school: Dürer glass'. in M. H. G. Hoffmann, J. Lenhard, F. Seeger (ed.). *Activity and Sign – Grounding Mathematics Education. Festschrift for Michael Otte*, © 2003 Kluwer Academic Publishers. Printed in the Netherlands.
- Bartolini Bussi, M. G.: (to appear), 'The Meaning of Conics: historical and didactical dimension', in Hoyles C., Kilpatrick J. & Skovsmose O. (eds.), *Meaning in Mathematics Education*, Kluwer Academic Publishers.
- Bartolini Bussi M. G., Boni M. & Ferri F.: (to appear), 'Construction problems in primary school: a case from the geometry of circle', in Boero P. (Ed.), *Theorems in School from History and Epistemology to Cognitive and Educational Issues*, Dordrecht: Kluwer Academic Publishers.
- Bernardi C.: 2003, 'Un approccio teorico ai software geometrici: gruppi di trasformazioni e operazioni geometriche elementari' in *Proceedings of the National Conference 'Apprendere la matematica con le tecnologie'* (Gela, October 2003), Ghisetti e Corvi Editori
- Betti, B. & Canalini, R.: 2002, 'Abaco e notazione posizionale: storie di internalizzazione', in Malara N. A., Marchini C., Navarra G. & Tortora R. (eds.), *Processi didattici innovativi per la matematica nella scuola dell'obbligo: Studi ed esperienze con insegnanti e nelle classi*, 17-28, Pitagora Editrice Bologna.

- Bonotto, C.: 2001, 'How to connect school mathematics with students' out-of-school knowledge', *Zentralblatt für Didaktik der Mathematik*, 33 (3), 2001, 75-84
- Bonotto C.: 2003 Investigating the Mathematics incorporated in the real world as a starting point for mathematics classroom activities, in N.A. Pateman, B.J. Dougherty & J.T. Zilliox (eds.), *Proceedings of PME 27*, Honolulu, Hawai'i, 2, 129-136.
- Bonotto C., Basso M., Baccarin R. & Feltresi M, 2002 'Studi esplorativi sulla comprensione del concetto di numero decimale attraverso l'uso del righello' N. Malara, C. Marchini, G. Navarra, R. Tortora (eds) *Processi didattici innovativi per la matematica nella scuola dell'obbligo*, Pitagora, Bologna, 2002, 43-58
- Bonotto, C. & Ceroni, G.: 2003, 'How can the use of suitable cultural artifacts as didactic materials facilitate and make more effective mathematics learning?', *Proceedings of CIEAEM 55*, Plock (Poland), 22 – 28 July 2003, 45-47
- Bottino, R. M.: 2000 'Advanced Learning Environments: Changed Views and Future Perspectives', in M. Ortega & J. Bravo (eds), *Computers And Education: Towards An Interconnected Society*, The Netherlands, Dordrecht: Kluwer Academic Publishers, 11-27.
- Bottino, R. M. & Chiappini, G.: 2002, 'Advanced technology and learning environment', in Lyn D. English (ed), *Handbook of international research in mathematics education*, 757-786, Lawrence Erlbaum, Associates Publisher.
- Cerulli, M. & Mariotti, M. A.: 2003, 'Building theories: working in a microworld and writing the mathematical notebook', *Proceedings PME 27*, Honolulu, 181-188.
- Chiappini, G., Pedemonte, B. & Robotti, E.: 2003, 'Mathematical teaching and learning environment mediated by ICT', in C. Dowling & K-W. Lai (eds.), *Information and Communication Technology and the Teacher of the Future*, USA, Massachusetts, Norwell: Kluwer Academic Publishers.
- Chiappini, G. & Reggiani, M.: 2003, 'Toward a didactical practice based on mathematics laboratory activities', *Proceedings of Cerme 3 (Third Conference of the European Society for Research in Mathematics Education)*, Bellaria, Italy, 28 febbraio-3 marzo 2003.  
[http://www.dm.unipi.it/~didattica/CERME3/draft/proceedings\\_draft/TG9\\_draft/](http://www.dm.unipi.it/~didattica/CERME3/draft/proceedings_draft/TG9_draft/)
- De Petro C., Margarone D., Micale B., Petrone A.: 2003, 'Un modello di formazione e l'insegnamento della geometria', *La Matematica e la sua Didattica*, to appear.
- Facenda A.M., Fulgenzi P., Nardi J., Paternoster F.: 2002, 'Rapporto tra disegno e modello dinamico nella costruzione delle immagini mentali', in Malara N. A., Marchini C., Navarra G. & Tortora R. (eds.), *Processi didattici innovativi per la matematica nella scuola dell'obbligo: Studi ed esperienze con insegnanti e nelle classi*, 83-101, Pitagora Editrice Bologna.
- Ferrara, F. & Robutti, O.: 2002, 'A graphical approach to functions through Body Motion', L Bazzini & C. Whybrow Inchley (eds.), *Proceedings of CIEAEM 53*, Verbania, Italy, 321-326.
- Ferrara, F. & Robutti, O.: 2002, 'Approaching graphs with motion experiences', A. D. Cockbrun & E. Nardi (eds.), *Proceedings of PME 26*, 4, 121-128.
- Ferri, F.: 2002, 'L'abaco e lo zero', in Malara N. A., Marchini C., Navarra G. & Tortora R. (eds.), *Processi didattici innovativi per la matematica nella scuola dell'obbligo: Studi ed esperienze con insegnanti e nelle classi*, 103-114, Pitagora Editrice Bologna.
- Formica D., Jacona D., Lo Cicero A., Margione D., Milone C., Mirabella A.: 2001, 'Uno strumento per l'analisi critica dei libri di testo', *L'insegnamento della matematica e delle scienze integrate*, 24A, 2, 129-144.
- Furinghetti, F., Olivero, F. & Paola, D.: 2001, 'Students approaching proof through conjectures: snapshots in a classroom', *International journal of Mathematical Education in Science and Technology*, vol.32, n.3, 319-335.
- Galoppin P. & Zuccheri L.: 2002, 'A didactical experience carried out using at the same time two different tools: a conceptual one and a technological one', in: Novotna J. (ed.), *CERME2 Proceedings (European Research in Mathematics Education II)*, Mariánské Lázně, Czech Republic, February 24-27, 2001), 2002, Charles University, Faculty of Education, Praga, 152-

162, [ISBN 80-7290-075-7]

- Impedovo, M.: 2002, 'The NT (New Technology) Hypothesis', *ICTM2 Proceedings* (Crete, July 2002).
- Iozzi, F.: 2002, 'Collaboration and assessment in a technological framework', *ICTM2 Proceedings* (Crete, July 2002).
- Laborde, C. & Mariotti, M. A.: 2002, 'Grounding the notion of function and graph in DGS', *Actes de CabriWorld 2001* – Montreal.
- Lemut E., 2003, 'Software Environments supporting and enhancing Systemic Thinking', *Proceedings of Cerme 3 (Third Conference of the European Society for Research in Mathematics Education)*, Bellaria, Italy, 28 febbraio-3 marzo 2003.  
[http://www.dm.unipi.it/~didattica/CERME3/draft/proceedings\\_draft/TG9\\_draft/](http://www.dm.unipi.it/~didattica/CERME3/draft/proceedings_draft/TG9_draft/)
- Mariotti, M.: 2001, 'Justifying and proving in the Cabri environment', *International Journal of Computer for Mathematical Learning*, Dordrecht: Kluwer, 6(3), 257-281
- Mariotti, M.: 2001, 'Introduction to proof: the mediation of a dynamic software environment', (Special issue) *Educational Studies in Mathematics* Volume 44, Issues 1&2, Dordrecht: Kluwer, 25 – 53.
- Mariotti, M. & Cerulli, M.: 2001, 'Semiotic mediation for algebra teaching and learning', *Proceedings of the 25<sup>th</sup> PME Conference*, The Netherlands, 3, 343-350
- Mariotti, M.: 2002, 'Influence of technologies advances on students' math learning', in English, L. et al. (eds.), *Handbook of International Research in Mathematics Education*, Lawrence Erlbaum Associates, 695-723.
- Mariotti, M. & Cerulli, M.: 2002, 'L'algebrista : un micromonde pour l'enseignement et l'apprentissage de l'algèbre de calcul', *Sciences et Techniques Educatives*, Numéro spécial Algèbre vol 9 n°1-2, 149-170.
- Mariotti, M., Laborde, C. & Falcade, R.: 2003, 'Function and graph in DGS environment', *Proceedings PME 27*, 237-244.
- Maschietto, M.: 2002, 'The transition from algebra to analysis: the use of metaphors in a graphic calculator environment', in J. Novotná (ed), *Proceedings of CERME 2*, vol. II, Charles University, Czech Republic, 542-553.
- Olivero, F.: 2001, 'Conjecturing in open geometric situations in a dynamic geometry environment: an exploratory classroom experiment', in C. Morgan & K. Jones (eds.), *Research in Mathematics Education*, London, vol.3, 229-246. ISBN 0-9538498-1-3.
- Olivero, F., Paola, D. & Robutti, O.: 2001, 'Avvio al pensiero teorico in un ambiente di geometria dinamica', *L'Educazione Matematica*, XXII, serie VI, vol.3, 127-148.
- Olivero, F. & Robutti, O.: 2001, 'An exploratory study of students' measurement activity in a dynamic geometry environment', *Proceedings of CERME 2*.
- Olivero, F. & Robutti, O.: 2001, 'Measures in Cabri as a bridge between perception and theory', M. Van den Heuvel-Panhuizen (ed.), *Proceedings of PME 25*, 4, 9-16.
- Olivero, F., Paola, D. & Robutti, O.: 2002, 'Teaching proof in a dynamic geometry environment: what mediation?', in L. Bazzini & C. Whybrow Inchley (eds.), *Proceedings of CIEAEM 53*, Verbania, Italy, 307-312.
- Olivero, F. & Robutti, O.: 2002, 'How much does Cabri do the work for the students?', in A. D. Cockburn & E. Nardi (eds.), *Proceedings of PME 26*, 4, 9-16.
- Olivero, F.: 2003, 'Cabri as a shared workspace within the proving process', in N.A. Pateman, B.J. Dougherty & J. Zilliox (eds.), *Proceedings of the 2003 Joint Meeting of PME and PMENA*, Honolulu, Hawaii, vol.3, pp.429-436. ISSN 0771-100X
- Osimo, G.: 2002, 'E-learning in Mathematics undergraduate courses (an Italian experience)', *ICTM2 Proceedings* (Crete, July 2002).
- Pergola, M., Zanolini, C., Martinez, A., Turrini, M.: 2001, 'Modelli fisici per la matematica: sulle sezioni del cilindro retto', *Progetto Alice*, vol. II, n° 4, 143-150.

- Pergola, M., Zanolì, C., Martínez, A., Turrini, M.: 2002, 'Modelli fisici per la matematica: parallelogrammi, antiparallelogrammi e deltoidi articolati', *Progetto Alice*, vol. III, n° 8, 323-346.
- Pergola, M., Zanolì, C., Martínez, A., Turrini, M.: 2002, 'Modelli fisici per la matematica: conigrafi flessibili', *Progetto Alice*, vol. III, n° 9, 559-571.
- Pergola, M., Maschietto, M., 'Modelli fisici per la matematica: biellismi del Peaucellier e del Delaunay', *Progetto Alice*, vol. IV, n° 11.
- Pesci, A.: 2000, 'The properties of necessity and sufficiency in the construction of geometric figures with Cabri', *Proceedings PME 24*, Hyoshima, 23-27 July 2000, Vol. 4, 73-80.
- Reggiani, M.: 2000, 'Graphic representation and algebraic expressions: an example of software mediation', in Rogerson A. (ed), *Proceedings of the International Conference on Mathematics Education into the 21st Century: Mathematics for Living*, Amman (Jordan), 251-256
- Reggiani, M.: 2002, 'Scrivere, leggere ed elaborare espressioni algebriche con DERIVE' (Writing, reading and processing algebraic expressions with Derive), SFIDA XIV "La scrittura in algebra", in Drouhard, Maurel (eds.), Séminaire Franco-Italien de Didactique de l'Algèbre, Actes des Séminaires, SFIDA 13 à SFIDA 16, vol.IV, 1999 - 2001, IREM de Nice, XIV 23-27
- Reggiani, M.: 2002, 'Arithmetic, algebra and technology: a study on beginner pupils', in Rogerson A. (ed.), *Proceedings of The International Conference The Humanistic Renaissance in Mathematics Education*, 20-25/9/2002, Palermo, 312-316
- Robutti, O.: 2003, 'Real and virtual calculator: from measurement to definite integral', *Proceedings of CERME 3*, Bellaria, Italy.  
[http://www.dm.unipi.it/~didattica/CERME3/draft/proceedings\\_draft/TG9\\_draft/](http://www.dm.unipi.it/~didattica/CERME3/draft/proceedings_draft/TG9_draft/)
- Robutti, O. & Sabena, C.: 2003, 'La costruzione del significato di integrale. Parte A', *L'insegnamento della matematica e delle scienze integrate*, Vol. 26B n.4, 473-498.
- Rocco M., 2000 'Cabri in un tirocinio per studenti stranieri', A.Andronico, G.Casadei, G.Sacerdoti (editors), 2000, *DIDAMATICA2000-Informatica per la didattica.- Atti, Vol.2- Esperienze*, Società editrice "Il Ponte Vecchio", 203-209
- Zuccheri, L.: 2003, Problems arising in teachers' education in the use of didactical tools, to appear in: *CERME3 Proceedings (European Research in Mathematics Education III*, Bellaria, February 28 – March 3, 2003),  
[http://www.dm.unipi.it/~didattica/CERME3/draft/proceedings\\_draft/TG9\\_draft/](http://www.dm.unipi.it/~didattica/CERME3/draft/proceedings_draft/TG9_draft/)